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PHYSICS OF WAVE PROPAGATION IN ~~WEST~~ ALLUVIUM:
A PROGRESS REPORT

James G. Berryman
and
Lewis Thigpen

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PHYSICS OF WAVE PROPAGATION IN WET ALLUVIUM: A PROGRESS REPORT

JAMES G. BERRYMAN AND LEWIS THIGPEN

Lawrence Livermore National Laboratory

P. O. Box 808, L-201

Livermore, CA 94550

ABSTRACT

In 1983, a long-term research project was initiated with the immediate goal being improved understanding of large amplitude stress waves in wet and dry porous materials and the ultimate goal being improved predictive capabilities for containment applications. A comprehensive theory of wave propagation in partially liquid-saturated porous materials has been developed. Some of the consequences of this theory have been explored; for example, the theory predicts compressional and shear wave speeds for partially saturated laboratory samples in agreement with experiment at lower (seismological) frequencies. The theory also shows that, by using only one (common) assumption (i.e., capillary pressure effects are neglected), code calculations of partial saturation problems may be reduced to computations no more complicated than those of full saturation problems.

INTRODUCTION

We wish to take this opportunity to present an overview of our progress to date on understanding the physics of wave propagation in wet porous materials. The discussion will be limited mainly to summarizing work of the authors since this project began in 1983. However, in addition, we will provide some insight into the prospects for this research to make a significant impact on containment calculations in the future.

One discussion which provides motivation for this work was presented by B. W. Smith¹ at the Monterey Containment Symposium. Computer modelling of Nevada Test Site (NTS) alluvium is severely hindered by the lack of adequate laboratory and field experimental data on the constitutive behavior of the materials of interest. The observed behavior of the available laboratory samples can, of course, be modelled – using some favored phenomenological model (or a table look up procedure) – for the limited number of loading and unloading cycles which are typically studied for such samples. However, little can be done with certainty when the computed loading/unloading paths differ significantly from the typical ones or when samples are not available from the test site.² The computer modelling effort is therefore severely limited by the modeller's inability to obtain "complete" data sets to describe the *in situ* behavior of the materials at the site. Furthermore, as long as budgets are limited and timetables short, this lack of "adequate" data may be expected to continue. In order to break this deadlock, a new approach based on a detailed (microscopic) theory of the deformation behavior of granular/porous materials is clearly required. Such a theory should be expected to relate the microscopic mechanics of pore deformation and closure to the parameters in a computer model. Furthermore, if the theory is really successful (i.e., it has some real predictive power), it should allow the modeller to have greater confidence in the results of the calculations even though no more data is available than before.

The task of obtaining a general understanding of the deformation behavior of porous earth with saturating or only partially saturating pore fluids is daunting (to say the least). To make a start on the problem, we break it into smaller subtasks that can be attacked separately. Although some of the subtasks *must* be done in series – depending as they do on the results of other subtasks, many may also be done in parallel. The two main categories of study are: (1) reversible and (2) irreversible deformations. For each of these categories, we need (1) to derive the general forms of the equations of motion and (2) to determine the values of the coefficients (often but not always constant) appearing in these equations. Furthermore, we need to seek confirming experimental evidence both for the form of the equations and the values of the coefficients. Although it is clear that the deformations of most interest in containment calculations are the irreversible ones leading to permanent compaction and closure of pores, it is nevertheless essential to understand the reversible

deformations first: not only because (1) the reversible phenomena are relatively easy to understand since energy methods may be applied, but also because (2) the pre-yield loading and post-yield unloading paths for the irreversible deformations are themselves strongly dependent on the reversible behavior of the granular/porous medium. The reversible deformations have therefore been given the most attention in the early phases of this effort and most of the results described in the remainder of this paper refer to deformations for which energy methods apply.

ENERGY METHODS AND EQUATIONS OF MOTION

When the mechanical and thermodynamical processes set in motion by a deformation are reversible, an energy functional which includes all the important effects involved in the motion may be constructed. Equations of motion may then be found by an application of Hamilton's principle. Such variational methods based on energy functionals are well-known in continuum mechanics. Thus, the only really new feature in the present context is the degree of complexity; porous earth may be composed of many types of solid constituents and the pore space may be filled with a mixture of water and air. Some irreversible effects may also be included in the variational method (e.g., losses of energy due to drag between constituents) when they may be analyzed in terms of a dissipation functional. Other irreversible effects such as those associated with collapse of pore space lie beyond the scope of the traditional variational approaches; the forms normally used for the energy functionals are quadratic *with constant coefficients* in the linear problems or simply positive definite polynomials *with constant coefficients* for nonlinear problems. During pore collapse, the values of the "constant coefficients" in the energy functionals are changing so the usefulness of the variational method is decreased.

We will not get into specifics here, but the authors have constructed a quite general Lagrangian variational principle³ for nonlinear and semilinear deformations (largely reversible) of dry and fluid saturated porous solids. This approach is very closely related to an Eulerian variational formulation by Drumheller and Bedford⁴ for flow of complex mixtures of fluids and solids. We have shown that our theory reduces correctly to Biot's equations of poroelasticity⁵ for small amplitude wave propagation and that it also reduces correctly to Biot's theory⁶ of nonlinear and semilinear rheology for porous solids when the deformations are sufficiently slow. The resulting theory is a nontrivial generalization of Biot's ideas including explicit equations of motion for changes of solid and fluid density. Furthermore, if we assume that capillary pressure effects may be neglected, then the theory also shows that calculations on problems with only partially saturated pores may be reduced to computations of the same level of difficulty as those for fully saturated pores.⁷ We expect the general theory to give a very good account of the behavior of wet porous

materials during elastic (reversible) deformations. Calculations using the theory are in progress and will be reported elsewhere.

CONSTANTS FOR PARTIAL SATURATION AND HETEROGENEITY

Although the energy methods just described provide a very satisfactory technique for deriving the form of the equations of motion, some other approach is required to obtain the various coefficients which appear in the equations. Of course, if appropriate laboratory data were available, then no further theory would be required. However, when the needed data are lacking, a deeper analysis of the physical constants in these problems is required. Again the main difficulty with obtaining theoretical estimates for real earth materials is the complex structure of these materials. A theory of composites is called for. The theory which we have applied to this problem is a self-consistent effective medium theory which has been shown to give good predictions for electrical conductivities and elastic constants for two-phase composites. In the present class of problems, the two phases are typically a solid constituent and the void space. However, these theories are not restricted to two phase composites and therefore can be used quite effectively in studies of rocks. These methods have been used to find estimates of the coefficients for wave propagation in porous media with multiple pore fluids (i.e., partially saturated porous media) and with multiple solid constituents (i.e., heterogeneous porous media). In both of these problems, the resulting coefficients serve to generalize Biot's theory of poroelasticity beyond its originally rather restrictive sphere of applicability.

First, consider a porous medium (e.g., a rock) composed of a variety of solid constituents. An appropriate generalization of Gassmann's equation must be found for materials heterogeneous on the microscopic scale. The form of this generalization has already been studied by Brown and Korringa.⁸ Their main result may be expressed as

$$H - \frac{4}{3}\mu = K + \sigma C, \quad (1)$$

$$C = \sigma / \left[\frac{\sigma}{K_s} + \phi \left(\frac{1}{K_f} - \frac{1}{K_\phi} \right) \right], \quad (2)$$

$$M = C / \sigma, \quad (3)$$

where

$$\sigma = 1 - K / K_s. \quad (4)$$

The constants H , C , and M are coefficients in Biot's strain-energy functional for an isotropic, linear porous medium saturated with fluid. This quadratic functional of the solid elastic strain invariants e , I_2 , and of the increment of fluid content ζ has the form

$$2W = He^2 - 2C\epsilon\zeta + M\zeta^2 - 4\mu I_2. \quad (5)$$

The constants appearing in Eqs.(1)-(4) are the porosity ϕ and shear modulus μ of the porous frame, the bulk modulus of the pore fluid K_f , and three other bulk moduli characteristic of the porous frame:

$$\frac{1}{K} = -\frac{1}{V} \left(\frac{\partial V}{\partial p_d} \right)_{p_f}, \quad (6)$$

$$\frac{1}{K_s} = -\frac{1}{V} \left(\frac{\partial V}{\partial p_f} \right)_{p_d}, \quad (7)$$

and

$$\frac{1}{K_\phi} = -\frac{1}{V_\phi} \left(\frac{\partial V_\phi}{\partial p_f} \right)_{p_d}, \quad (8)$$

where V is the total volume, $V_\phi = \phi V$ is the pore volume, p is the external pressure, p_f is the pore pressure, and $p_d = p - p_f$ is the differential pressure. Brown and Korrington⁸ point out that, although these three bulk moduli have simple physical interpretations, this “does not necessarily help in knowing their values.” Actually the constant K is just the dry frame bulk modulus and has been studied extensively. However, the values of the remaining constants K_s and K_ϕ are not known unless the porous frame is homogeneous on the microscopic scale in which case $K_s = K_\phi = K_m$, the bulk modulus of the constituent material.

A method of finding estimates of the constants C^* and σ^* in Eqs.(1)-(4) directly based on an effective medium approximation has been presented recently.⁹ The expressions for C^* and σ^* are found to be

$$C^* = \sigma^* / \left[\left\langle \frac{1}{M(\bar{x})} \right\rangle + \left\langle \frac{\sigma^2(\bar{x}) - (\sigma^*)^2}{K(\bar{x}) + \frac{4}{3}\mu^*} \right\rangle \right] \quad (9)$$

and

$$\sigma^* = \left\langle \frac{\sigma(\bar{x})}{K(\bar{x}) + \frac{4}{3}\mu^*} \right\rangle / \left\langle \frac{1}{K(\bar{x}) + \frac{4}{3}\mu^*} \right\rangle. \quad (10)$$

The averages $\langle \cdot \rangle$ in (9) and (10) are spatial (\bar{x}) averages. The constant μ^* is an estimate of the effective shear modulus of the dry porous frame. Notice that (10) does not depend on C^* ; therefore, both constants have values determined explicitly by the formulas. These results have been shown to be fully consistent with the general form derived by Brown and Korrington and the numerical predictions of these formulas have also been shown to be quite reasonable.⁹ The main conclusion of this analysis is that the presence of a small amount of low modulus material can substantially decrease the effective solid moduli in Gassmann's equations.

Besides the usual restriction to porous media composed of a single granular material, Biot's (linear) equations of poroelasticity have several other limitations. The equations

were derived with an explicit long-wavelength assumption and also with strong implicit assumptions of homogeneity and isotropy on the macroscopic scale. Another restriction assumes that the pore fluid is uniform and that it fully saturates the pore space. If the pore fluid fills only part of the connected pore space or if the pores are filled with more than one kind of fluid, then Biot's equations are not adequate to describe all the possible modes of oscillation of the system.⁷ However, at long wavelengths (consistent with the derivation of the equations), it is possible to treat systems with multiple pore fluids within the context of Biot's equations. Indeed, the authors have shown elsewhere^{10,11} how to construct another effective medium approximation for partially saturated materials. Important results which may be derived this way are the effective bulk modulus and density of a composite fluid occupying the pores of a material well-described by Biot's equations of poroelasticity thus generalized for partial saturation.

If the total pore volume fraction ϕ is occupied by two fluids (say a liquid and a gas) with bulk moduli K_l and K_g and volume fractions satisfying $\phi_l + \phi_g = \phi$, then we have shown that the effective fluid bulk modulus is given by

$$1/K_f^* = S_l/K_l + S_g/K_g, \quad (11)$$

where the saturation levels are defined as

$$S_l = \phi_l/\phi, S_g = \phi_g/\phi, \quad (12)$$

which is the well-known result sometimes called Wood's formula.¹² The effective fluid bulk modulus (11) is then the value to be used in Gassmann's equation [use Eqs.(1)-(3) with $K_s = K_\phi = K_m$]. A similar calculation for the effective fluid density gives the simple result that

$$\rho_f^* = \phi_l \rho_l + \phi_g \rho_g, \quad (13)$$

i.e., the effective density is just the average density.

To check the results of the theory against experiment, we have calculated the fast compressional and shear wave speeds v_+ and v_s using our theory of partial saturation. Figures 1 and 2 show the results obtained when the formulas (11) and (13) are used to estimate the fluid bulk modulus and density needed in the equations of poroelasticity. The experimental data on Massillon sandstone are taken from Murphy.^{13,14} The parameters used in the calculations were $K = 1.02 \text{ GPa}$, $\mu = 1.44 \text{ GPa}$, $K_s = 35.0 \text{ GPa}$, $\mu_s = 25.0 \text{ GPa}$, $\rho_s = 2.66 \text{ g/cc}$, $K_l = 2.25 \text{ GPa}$, $\rho_l = 0.997 \text{ g/cc}$, $K_g = 1.45 \times 10^{-4} \text{ GPa}$, $\rho_g = 1.20 \times 10^{-3} \text{ g/cc}$, $\phi = 0.22$, $\kappa = 740 \text{ mD}$ (permeability), $h = 20 \text{ }\mu\text{m}$ (hydraulic radius), and $\omega = 2\pi \times 560 \text{ Hz}$ (frequency). The values of K and μ for the frame are chosen to fit the experimental results at $S_g = 0$. The remaining points of the theoretical curve (the solid lines) follow without further adjustment

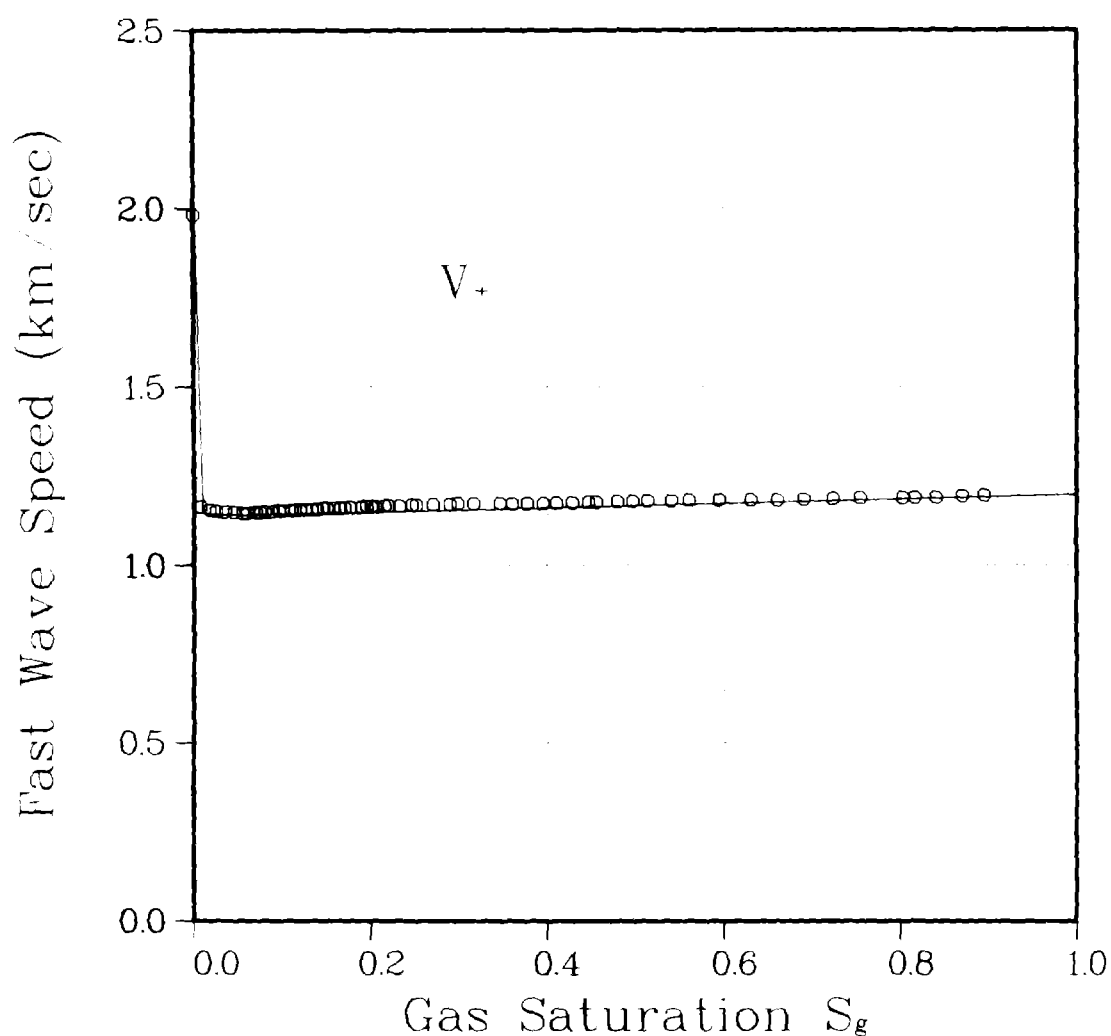


Figure 1. Fast compressional wave speed for a Massilon sandstone partially saturated with water versus the fraction of the porosity filled with gas S_g , i.e., the gas saturation. The solid line is the theoretical result obtained by combining Biot's theory of full saturation with the effective medium result for the effective bulk modulus of a liquid/gas mixture – Eqs. (11) and (13) in the text. Values of the parameters used are quoted in the text. The circles are the experimental data points of Murphy.^{13,14}

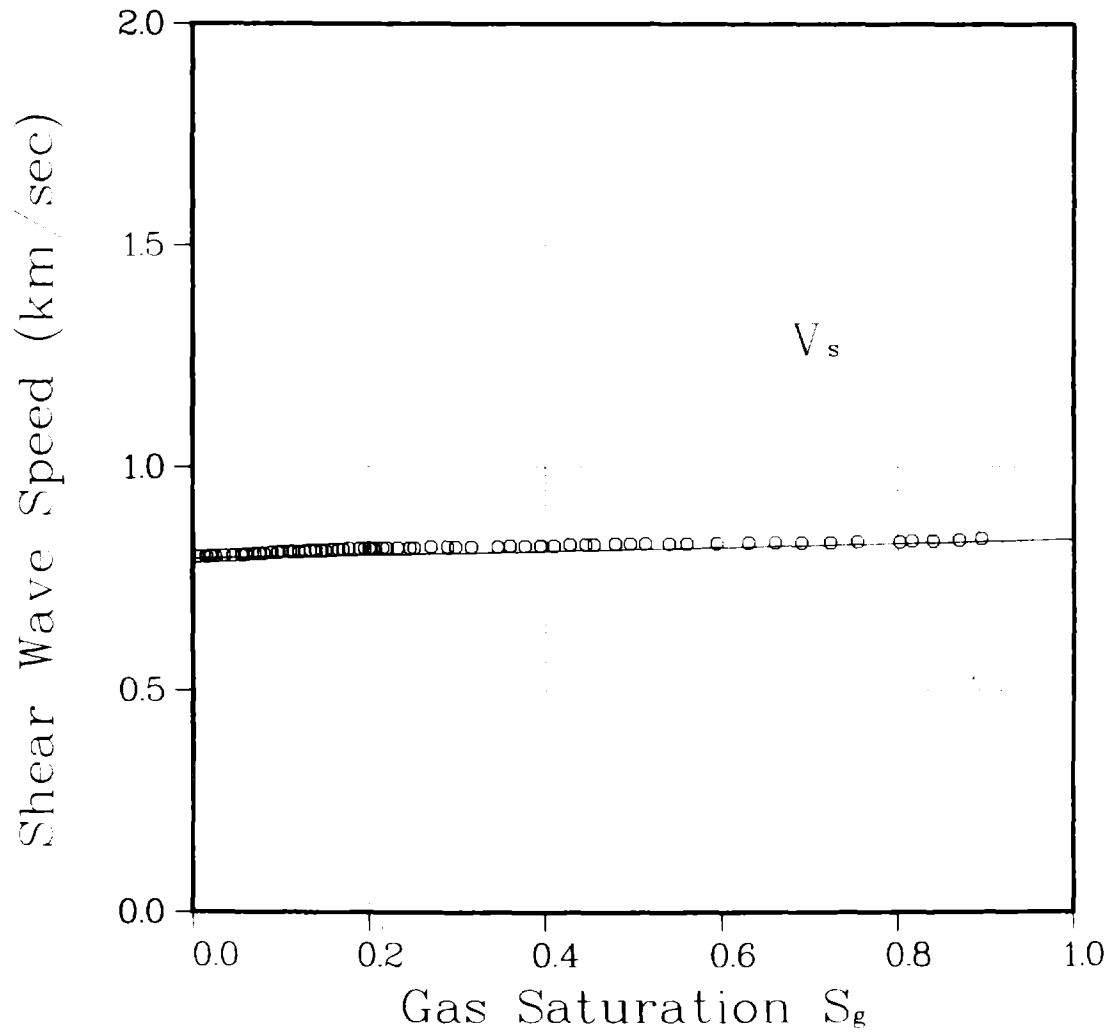


Figure 2. Shear wave speed versus gas saturation S_g for Massilon sandstone as in Figure 1. Experimental data courtesy of W. F. Murphy.^{13,14}

of parameters. The agreement between the theory and experiment is quite good for this example.

These results had been found previously using *ad hoc* arguments concerning the expected low frequency behavior of the composite fluid occupying the pores.^{13,14} The authors have shown elsewhere that our methods can be used to estimate other parameters in the problem such as the effective viscosity⁸ of the composite fluid. So this approach certainly has greater potential than a strictly phenomenological method to provide real insight into all aspects of this very difficult problem. The theory has therefore essentially been validated for lower frequency wave propagation through fluid saturated and partially saturated porous rocks.

Thus, we have found methods of estimating effective properties of complex porous materials composed of multiple solid constituents and multiple pore fluids. Considerable work remains of course to check these predictions against experiments and to generalize the results for studies of the dynamic behavior of such materials.

PORE COLLAPSE AND SHEAR STRENGTH

To simplify the following discussion, we will neglect effects due to changes of phase in the materials constituting the porous medium. Then, when pressure is applied to a dry granular/porous material, the initial (pre-yield) loading curve is reversible (i.e., describable by an energy functional) until some threshold value of the pressure (characteristic of the loading path and the material) is reached. Above this threshold pressure, a granular material will begin to yield by suffering irreversible loss of pore space.¹⁵ When the applied pressure is gradually released, the (post-yield) unloading curve is again reversible (i.e., describable by a new energy functional) until reloading brings the magnitude of the applied pressure again to its highest previous value (assuming the nature of the loading is unchanged). When this new pressure threshold is reached, still more pore space may be irreversibly crushed out of the material. (The presence of pore water has little mechanical effect on this process unless the remaining connected pore space is filled with liquid; then, the effective pressure must be used in the analysis instead of the applied pressure.) Thus, even though a single energy functional cannot be used to analyze the behavior of the granular/porous material on its yield surface, an array of such energy functionals parameterized by the current value of the porosity can be used virtually everywhere off the yield surface. Furthermore, if these energy functionals change monotonically with the porosity, then several things can be said about the flow rules governing the pore collapse.

To illustrate the preceding discussion, consider the problem of the expansion of a

spherical shell of granular/porous material. It can be shown that, in this geometry, the elastic stress energy functional may be written in terms of the principal stresses as

$$W = (\tau_{rr} + 2\tau_{\theta\theta})^2/18K(\phi) + (\tau_{rr} - \tau_{\theta\theta})^2/6\mu(\phi) \quad (14)$$

where τ_{rr} and $\tau_{\theta\theta} = \tau_{\phi\phi}$ are the radial and hoop stresses (positive in tension) respectively. $K(\phi), \mu(\phi)$ are the bulk and shear moduli of the material which are assumed to be monotonically increasing functions of the current value of the porosity ϕ . Following the analysis of Hill¹⁶ for the elastic/plastic problem, we find that if the initial inner and outer radii of the spherical shell are a_0 and b_0 then the stresses before yield are given by

$$\tau_{rr} = -p(\frac{b_0^3}{r^3} - 1)/(\frac{b_0^3}{a_0^3} - 1), \quad (15)$$

$$\tau_{\theta\theta} = p(\frac{b_0^3}{2r^3} - 1)/(\frac{b_0^3}{a_0^3} - 1). \quad (16)$$

Hill's discussion of the expanding spherical shell is for ductile metals; thus, he uses the maximum shear theory of failure¹⁷ to provide his yield criterion

$$\tau_{\theta\theta} - \tau_{rr} = Y, \quad (17)$$

which is either the Tresca or von Mises criterion since they are equivalent for the present problem. Since rocks tend to be much more brittle than metals¹⁸ and since the initial failure mechanism for this problem is expected to be a failure of the inner surface in tension [since Eq.(16) is positive], Eq.(17) may not be the best choice of failure criterion. However, we may still make some definite statements about the problem regardless of the explicit failure criterion assumed.

Now, even though the changing state of the material during pore collapse is not describable by an energy functional, there are still several physical quantities about which general quantitative statements can be made. For example, for slow deformations $\tau_{rr}, \tau_{\theta\theta}$ satisfy equilibrium stress equations and the radial stress τ_{rr} must clearly be continuous everywhere and at all times. Furthermore, we expect the elastic energy stored in the material prior to the pore collapse to remain stored and therefore recoverable. If there is some strain hardening during pore collapse, then the amount of recoverable energy in the material will increase monotonically. If we consider an element of a thick spherical shell ($b_0 \gg a_0$) on the inner cavity surface (i.e., $r = a_0$), then the elastic energy of such an element prior to collapse is

$$W_0 = (\tau_{rr} - \tau_{\theta\theta})^2/6\mu(\phi). \quad (18)$$

Thus, the stored energy upon subsequent unloading of the element is expected initially to satisfy

$$W_0 \leq W = (\tau_{rr} - \tau_{\theta\theta})^2/6\mu(\phi), \quad (19)$$

being careful now to distinguish between the elastic stress component $\tau_{\theta\theta}$ and the residual stress. For purposes of illustration, Fig. 3 plots the elastic energy W as a function of cavity pressure p for some porous material at various values of porosity ϕ . The solid lines correspond to $\phi = 0.30, 0.25, 0.20, \dots, 0.0$. The dot-dash line corresponds to the failure surface. We see that, as the porosity decreases, the (confined) shear modulus increases so the elastic energy at a fixed value of p decreases. Initial loading takes us up one of the solid curves during elastic deformation, then up the dot-dash line (for a strain hardening material) during pore collapse, until unloading brings us back to the origin along another solid curve corresponding to the compacted material of lower porosity.

As a consequence of (19), we find that one criterion for the yield point upon reloading of the material is

$$(\tau_{rr} - \tau_{\theta\theta})^2 \geq 6\mu(\phi)W_0, \quad (20)$$

which may be compared to (17). This result is very general – depending only on an assumption of conservation of elastic energy, not on any specific mechanism for the initial failure. Furthermore, it may provide some insight into the observed^{19,20} correlation between rock strength and shear modulus. The result is also closely related to the observation in ductile materials that the strain energy of distortion may be used as a criterion of yielding and failure instead of the maximum shear.¹⁷ Finally, these ideas may also be used to analyze the residual stress problem, but we will leave that discussion to a future publication.

CONCLUSIONS

Starting from a Lagrangian variational principle for porous media with density dependent microstructure very closely related to the Eulerian variational principle of Drumheller and Bedford,⁴ the authors have shown how to obtain equations of motion for various physically important problems in the rheology and wave propagation behavior of rocks, including: (1) Biot’s semilinear theory⁶ of rheology, (2) Biot’s linear theory⁵ of wave propagation in fluid-saturated porous media, (3) general linear equations for wave propagation in partially saturated porous media⁷. We have shown how to estimate the coefficients in these equations from the known physical properties of the material constituents, for both partially and fully saturated porous rocks^{10,11} and for rocks with a multiplicity of solid components.⁹ Furthermore, we have shown that closely related energy methods can provide some powerful clues to the behavior of granular media during large strain, inelastic deformation leading to the nonrecoverable crush out of pore space. Future work will concentrate on extending the theoretical methods to still more realistic models and to more comprehensive studies which validate the resulting predictions against laboratory data.

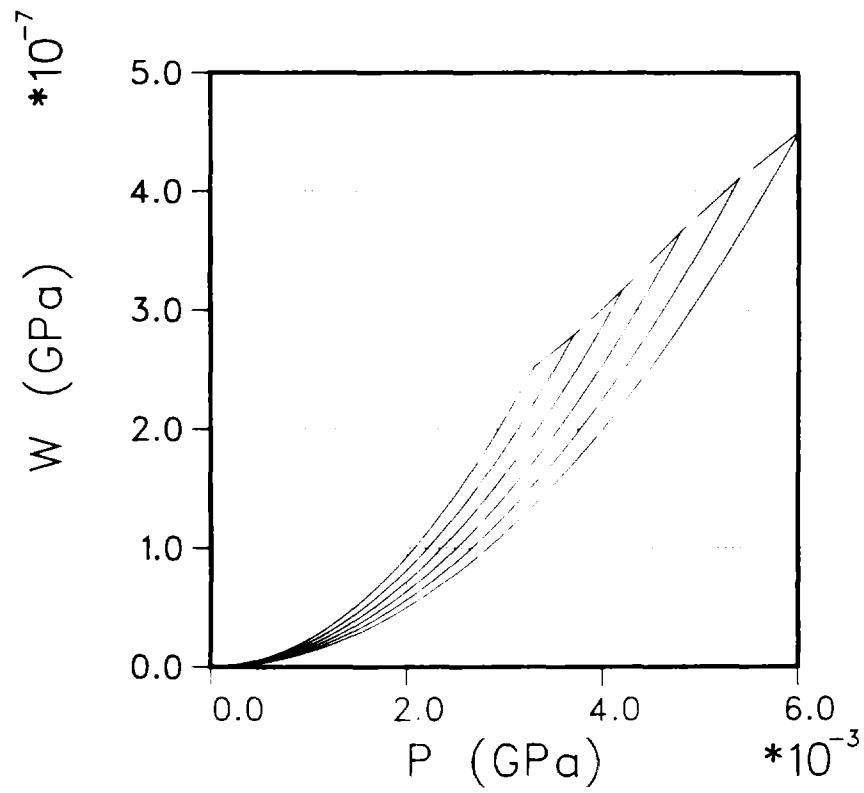


Figure 3. Plots of elastic energy W as a function of applied pressure p for a porous material at various values of porosity ϕ . The solid lines correspond to $\phi = 0.30, 0.25, 0.20, \dots, 0.0$. The dot-dash line corresponds to the failure surface.

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